

Genererende funktioner

Vi loader pakken **gfun**

```
> restart;with(gfun);
```

```
[Laplace, Parameters, algebraicsubs, algeqtodiffeq, algeqtoseries, algfuntoalgeq, borel,
cauchyproduct, diffeq*diffeq, diffeq+diffeq, diffeqtohomdiffeq, diffeqtorec, guesseqn,
guessgf, hadamardproduct, holexprtodiffeq, invborel, listtoalgeq, listtodiffeq,
listtohypergeom, listtolist, listtoratpoly, listtorec, listtoseries, poltodiffeq, poltorec,
ratpolytcoeff, rec*rec, rec+rec, rectodiffeq, rectohomrec, rectoproc, seriestoalgeq,
seriestodiffeq, seriestohypergeom, seriestolist, seriestoratpoly, seriestorec, seriestoseries]
```

(1)

Nogle lister

```
> L__1:=[1,-1,sqrt(2),27];
L__2:=[seq(1,n=0..20)];
L__3:=[seq(n,n=0..10)];
```

$$L_1 := [1, -1, \sqrt{2}, 27]$$

$$L_2 := [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$$

$$L_3 := [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$$

(2)

Kommandoen **listtoseries** hænger lister til tørre på potenserne

```
> listtoseries(L__1,x);
```

$$1 - x + \sqrt{2} x^2 + 27 x^3 + O(x^4)$$

(3)

```
> listtoseries(L__2,x);
```

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{15} + x^{16} + x^{17} + x^{18} + x^{19} + x^{20} + O(x^{21})$$

(4)

```
> listtoseries(L__3,x);
```

$$x + 2 x^2 + 3 x^3 + 4 x^4 + 5 x^5 + 6 x^6 + 7 x^7 + 8 x^8 + 9 x^9 + 10 x^{10} + O(x^{11})$$

(5)

Kommandoen **series** udregner generende funktioner på standardformen som et polynomium af i princippet uendelig grad, fx

```
> series((4)+(5),x,20);
```

```
series((4)*(5),x,20);
```

$$1 + 2 x + 3 x^2 + 4 x^3 + 5 x^4 + 6 x^5 + 7 x^6 + 8 x^7 + 9 x^8 + 10 x^9 + 11 x^{10} + O(x^{11})$$

$$x + 3 x^2 + 6 x^3 + 10 x^4 + 15 x^5 + 21 x^6 + 28 x^7 + 36 x^8 + 45 x^9 + 55 x^{10} + O(x^{11})$$

(6)

```
> series(1/(1-x),x,20);
```

```
series(1/(1-a*x),x,10);
```

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{15} + x^{16} + x^{17} + x^{18} + x^{19} + O(x^{20})$$

$$1 + a x + a^2 x^2 + a^3 x^3 + a^4 x^4 + a^5 x^5 + a^6 x^6 + a^7 x^7 + a^8 x^8 + a^9 x^9 + O(x^{10})$$

(7)

Bemærk syntaksen **series(genererende funktion, variabel, højeste grad man regner til)**

Kommandoen **seriestolist** giver den liste som en genererende funktion frembringer

```
> seriestolist((6)) ;
```

$$[0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55]$$
(8)

Man kan være interesseret i et enkelt element fra listen

```
> seriestolist((6)) [8] ;
```

$$28$$
(9)

► De generelle regneregler for genererende funktioner

► Fibonacci

▼ Hæveautomat

På hvor mange måder kan der udbetales 91 kr.?

Den generende funktion for udbetaling kun med 1-kroner er

```
> series(1/(1-x), x, 10) ;
```

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + O(x^{10})$$
(3.1)

Kun med 2-kroner

```
> series(1/(1-x^2), x, 10) ;
```

$$1 + x^2 + x^4 + x^6 + x^8 + O(x^{10})$$
(3.2)

Kun med 5-kroner

```
> series(1/(1-x^5), x, 10) ;
```

$$1 + x^5 + O(x^{10})$$
(3.3)

... kun med 50-kroner

```
> series(1/(1-x^50), x, 10) ;
```

$$1 + O(x^{50})$$
(3.4)

.... kun med 1000-kroner

```
> series(1/(1-x^1000), x, 10) ;
```

$$1 + O(x^{1000})$$
(3.5)

Så med alle mønt-/seddelenheder bliver det

```
> series(1/(1-x)/(1-x^2)/(1-x^5)/(1-x^10)/(1-x^20)/(1-x^50)/(1-x^100)/(1-x^200)/(1-x^500)/(1-x^1000), x, 100) ;
```

$$1 + x + 2x^2 + 2x^3 + 3x^4 + 4x^5 + 5x^6 + 6x^7 + 7x^8 + 8x^9 + 11x^{10} + 12x^{11} + 15x^{12} + 16x^{13} + 19x^{14} + 22x^{15} + 25x^{16} + 28x^{17} + 31x^{18} + 34x^{19} + 41x^{20} + 44x^{21} + 51x^{22} + 54x^{23} + 61x^{24} + 68x^{25} + 75x^{26} + 82x^{27} + 89x^{28} + 96x^{29} + 109x^{30} + 116x^{31} + 129x^{32} + 136x^{33} + 149x^{34} + 162x^{35} + 175x^{36} + 188x^{37} + 201x^{38} + 214x^{39} + 236x^{40} + 249x^{41} + 271x^{42} + 284x^{43} + 306x^{44} + 328x^{45} + 350x^{46} + 372x^{47} + 394x^{48} + 416x^{49} + 451x^{50} + 473x^{51} + 508x^{52} + 530x^{53} + 565x^{54} + 600x^{55} + 635x^{56} + 670x^{57} + 705x^{58} + 740x^{59} + 793x^{60} + 828x^{61} + 881x^{62} + 916x^{63} + 969x^{64} + 1022x^{65} + 1075x^{66} + 1128x^{67} + 1181x^{68} + 1234x^{69} + 1311x^{70} + 1364x^{71} + 1441x^{72} + 1494x^{73} + 1571x^{74} + 1648x^{75} + 1725x^{76} + 1802x^{77} + 1879x^{78} + 1956x^{79} + 2064x^{80} + 2141x^{81} + 2249x^{82} + 2326x^{83}$$
(3.6)

$$\begin{aligned}
 &+ 2434 x^{84} + 2542 x^{85} + 2650 x^{86} + 2758 x^{87} + 2866 x^{88} + 2974 x^{89} + 3121 x^{90} \\
 &+ 3229 x^{91} + 3376 x^{92} + 3484 x^{93} + 3631 x^{94} + 3778 x^{95} + 3925 x^{96} + 4072 x^{97} \\
 &+ 4219 x^{98} + 4366 x^{99} + O(x^{100})
 \end{aligned}$$

`> seriestolist((3.6) [92] ;`

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(3.7)

► Annuitet